DESIGN INITIATIVES FOR LEARNING: ICT AND GEOMETRY IN THE PRIMARY SCHOOL

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ABSTRACT

This paper draws on an ongoing project whose overall aim is to examine the ways in which new technologies can be used in educational settings to enhance learning. In particular it discusses the process of developing a design initiative for primary pupils to learn about polygons and their properties. The process draws explicitly on the role of the teacher within the context of exploiting the potential of a dynamic geometry environment. Design evolves in a contingent way related to pupils' developing conceptions and the purpose of the dynamic geometry environment is to help pupils pay attention to the invariant properties of particular quadrilaterals. Analysis of the data suggests that the pupils who participated in the mathematics design initiative learned important ideas related to quadrilaterals and their properties.

INTRODUCTION

This paper draws on an ongoing project whose overall aim is to examine the ways in which new technologies can be used in educational settings to enhance learning (www.interactiveeducation.ac.uk). The project is concerned with learning across a range of subjects (English, history, geography, mathematics, music, modern foreign languages, science), although this paper focuses on the mathematics education strand of the project. The research is framed by a socio cultural theory of learning which suggests that mental functioning of an individual has its origins in social life (Wertsch, 1991). This position stresses the crucial role which communication through language and other semiotic systems plays in learning and points to the importance of creating classroom environments which support the communication and exchange of ideas (Crook, 1994). Another implication of socio-cultural theory is the claim that human action is mediated by 'technical' and 'cognitive tools'. The notion of 'tools' includes a wide range of artefacts and semiotic systems, where "cultural artefacts are both material and symbolic; they regulate interactions with one's environment and oneself. In this respect they are 'tools' broadly conceived, and the master tool is language" (Cole & Engestrom, 1993, p. 9).

The project is based on working in partnership with teachers and researchers to design learning environments which are supported by research on teaching and learning. The teachers in the project were recruited through the establishment of partnership schools (3 primary, 5 secondary and one post-16 college), a relatively long and delicate process of negotiation with both teachers and senior management. In particular we wanted, as far as possible, to ensure that senior management teams did not impose involvement in the project on teachers.

The project is predicated on two assumptions. The first is that teachers are central to learning in schools and that much of previous research on the use of ICT for learning has underemphasised this crucial role (Sutherland & Balacheff, 1999). The second is that ICT should be incorporated into a designed learning situation as appropriate, with attention being paid to the whole classroom context to include classroom talk, work on paper and all the technologies that are normally available to a teacher.

PARTNERSHIP AND DESIGN

The mathematics design team consists of 14 teachers (from the primary, secondary and post-16 sectors), and three researchers (Godwin, Olivero and Sutherland). The team are working together over a period of two years both at University and in schools — fifteen days of teacher supply cover are allocated for each teacher partner. Working within the mathematics design team each teacher is responsible for designing a learning initiative for one class of students, which focuses on a key area within their subject which: a) pupils normally find difficult and b) could be enhanced by the incorporation of an appropriate ICT environment into the learning situation. Choosing this area was a process of negotiation between each teacher and the research team. This paper focuses on the work of one primary teacher Pat Peel, also a co-author of this paper. Before the third meeting of the mathematics design team (October 2001) all teachers had been asked to decide which group of students they planned to work with; which mathematical area they wanted to focus on; and which ICT environments they were thinking of using. Within the meeting they were asked to develop a concept map of this mathematical area, in order to focus on the mathematical domain and pupil's conceptions of this domain. Pat produced a concept map which focused on polygons, their properties and the associated mathematical language (e.g. parallel, perpendicular, rectangle). For several months after this meeting Pat investigated the potential of a range of ICT environments for supporting this area of learning, including the National Numeracy Strategy primary ICT pack. This led to a considerable amount of frustration and potential alienation from the project. The project researchers did not want to suggest using a dynamic geometry environment because at the first meeting of the mathematics design team (June 2001), when all teachers had worked with such an environment, the primary teachers had been relatively negative about Cabri and Pat had said that she thought that primary pupils would find the interface dull. However after rejecting a number of ICT environments and becoming increasingly critical of much of the software in the National Numeracy Strategy ICT pack (predominantly because of its non-mathematical nature) the project researchers decided to work on the development of a quadrilateral microworld (ie a small world in which pupils will 'bump into' the conceptual area of quadrilaterals) within Cabri Géomètre. Pat became interested in the potential of such a microworld and this was the starting point for working together on a mathematical design initiative which incorporated the use of Cabri.

The initial development process involved the project researchers working with Pat in her school for two afternoon and three after-school sessions over a period of several weeks. Between each session the project researchers continued to experiment with the quadrilateral microworld, wrote notes of the previous meeting and used these as a basis for ongoing development of the design initiative. As we said in the research proposal "design will be informed in an iterative way by theory, research-based evidence on the use of computers for learning, teacher's craft knowledge and the research team's expertise".

The design process involves working within the constraints of the situation in a creative and systematic way. For example the aim is that all pupils in Pat's Year 6 class (age 10-11) learn about quadrilaterals but within this constraint we have thought about: how to design a Cabri microworld which makes it likely that all pupils (whatever their confidence with language) will engage with both the properties of polygons and the names of these properties; how Pat will work with the class in the computer suite and use the interactive whiteboard and other technologies and what work pupils will do on paper and pencil in their normal classroom. Throughout this process Pat was learning about Cabri through hands-on work on her home computer with support from the research team.

The design process has also involved administering a diagnostic assessment to all pupils before and after the design initiative and interviewing 6 pupils, chosen to represent a spread of attainment. The design initiative reported in this paper was carried out between January 11th and March 1st 2002 and is being evaluated in a systematic way from the point of view of pupils' learning and how this relates to teaching.

DEVELOPING THE GEOMETRY DESIGN INITIATIVE

Pat works in an inner city muticultural primary school in Bristol, which is also part of an Education Action Zone (EAZ), a government funded initiative to boost educational standards in inner city schools. As part of this EAZ initiative the school was equipped with a networked computer room and interactive whiteboard in 2000. When the researchers first talked to Pat she said that her ultimate aim for joining the project was to improve the numeracy standards in her school.

As discussed already the theory which frames the project emphasises learners as active constructors of knowledge who come to a learning situation with a history of learning. It also foregrounds the social construction of knowledge and the role of language and other semiotic systems within this process. Within this context mathematics itself has also to be foregrounded. We also believe that pupils learn most effectively when they are supported to ask mathematical questions within a community of inquiry and when the teacher focuses their attention on the similarities and differences between mathematical objects (Brown & Coles, 2001). When working on a subject design we are loosely influenced by these theoretical ideas but do not and could not 'apply' them in a positivist way.

The designed mathematics learning environment developed by Pat emerged over a period of time. As discussed already one of the starting points for the design was Pat's concern with her pupils not being able to remember the names of geometrical shapes. Another starting point was the national Numeracy Strategy and the following excerpt was used as a guiding framework.

As outcomes Year 6 pupils should, for example

Name and begin to classify quadrilaterals; using criteria such as parallel sides, equal angles, equal sides, lines of symmetry..

Know properties such as:

- a parallelogram has its opposite sides equal and parallel;
- a rhombus is a parallelogram with four equal sides;
- a rectangle has four right angles and its opposite sides are equal;
- a square is a rectangle with four equal sides;
- a trapezium has one pair of opposite parallel sides;
- a kite has two pairs of adjacent sides equal;

Begin to know properties such as:

- the diagonals of any square, rhombus or kite intersect at right angles;
- the diagonals of any square, rectangle, rhombus or parallelogram bisect one another

Key Stage 2 National Numeracy Strategy, page 103

The 'intended' aim of this mathematics design initiative was for 10-11 year old pupils to learn:

1) to recognise particular polygons (quadrilaterals and triangles) and know the names of these figures;
2) to characterise geometrical shapes by their properties;
3) to classify figures hierarchically.

We chose to work with quadrilaterals as a starting point because we believe that these are the ones which are more common in everyday life and thus more likely to be familiar to pupils.

The results of the diagnostic assessment which was administered to all pupils before teaching started, and interviews with three pairs of pupils indicated that before starting the work the majority of the pupils could name all of the special quadrilaterals apart from a trapezium. However the majority did not know the properties of these geometrical figures, although some could recognise the property ‘opposite sides equal’ with respect to a rectangle. Some of the pupils interviewed said that parallel lines are like ‘train tracks’ which never cross (in response to a question about whether two lines which crossed were parallel one pupils said “no because if a train went along those tracks it would crash”). This image sometimes broke down with respect to the opposite sides of a rectangle. The longer pair of sides were said to be parallel but the shorter pair were said to be too far apart to be parallel. None of the pupils appeared to be able to make sense of perpendicularity and the majority of pupils could not identify an equilateral triangle from images of six triangles.

In summary it would appear that the majority of pupils in the class were recognising figures as visual gestalts, considered to be level 1 of van Hiele’s levels “In identifying figures they often use visual prototypes. Students say that a given figure is a rectangle for instance, because ‘it looks like a door’. They do not however, attend to geometric properties or to characteristic traits of the class of figures represented. That is, although figures are characterised by their properties, students at this level are not conscious of the properties” (Clements & Battista, 1992, page 427).

The design initiative consisted of 6 one hour lessons (as part of the numeracy hour) spread over eight weeks (because of a gap for ‘half term’ and one lesson where the teacher was sick). The work took place in a computer room where the pupils sometimes worked as a whole group with an interactive whiteboard, a flip chart and other visual aids, and sometimes worked in pairs at the computer. The following is an overview of the focus of each session. Only the first two sessions were planned in detail from the beginning and the remaining sessions were planned in the time between sessions.

Session 1 — playing with the quadrilateral microworld, introduce idea of properties

Session 2 — noticing and writing about properties of quadrilaterals

Session 3 — investigate diagonals of quadrilaterals (use Cabri measure tool)

Session 4 — investigate parallel and perpendicular lines

Session 5 — playing with triangle microworld and investigate properties of triangles

Session 6 — black box game, three questions to guess which quadrilateral/triangle is in my box; pupils construct rectangle or parallelogram in Cabri

All sessions centred around microworlds which were developed by the research team in collaboration with Pat (see Fig 1) and were aimed at supporting pupils to focus their attention on the invariant properties of particular polygons. The purpose of using a dynamic geometry environment was to allow pupils to manipulate for themselves, and 'see' mathematical properties, and the role of the teacher was initially to provoke pupils to become aware of what they were 'seeing' through spoken and written language. We customised the Cabri drop-down menus for the purpose of this study, restricting them to: point, line, segment, circle, perpendicular line, parallel line, reflection, parallel and perpendicular (in the check property menu), distance & length, label, comments, colour, fill). This restriction of the Cabri menus was aimed at focusing pupils' attention.

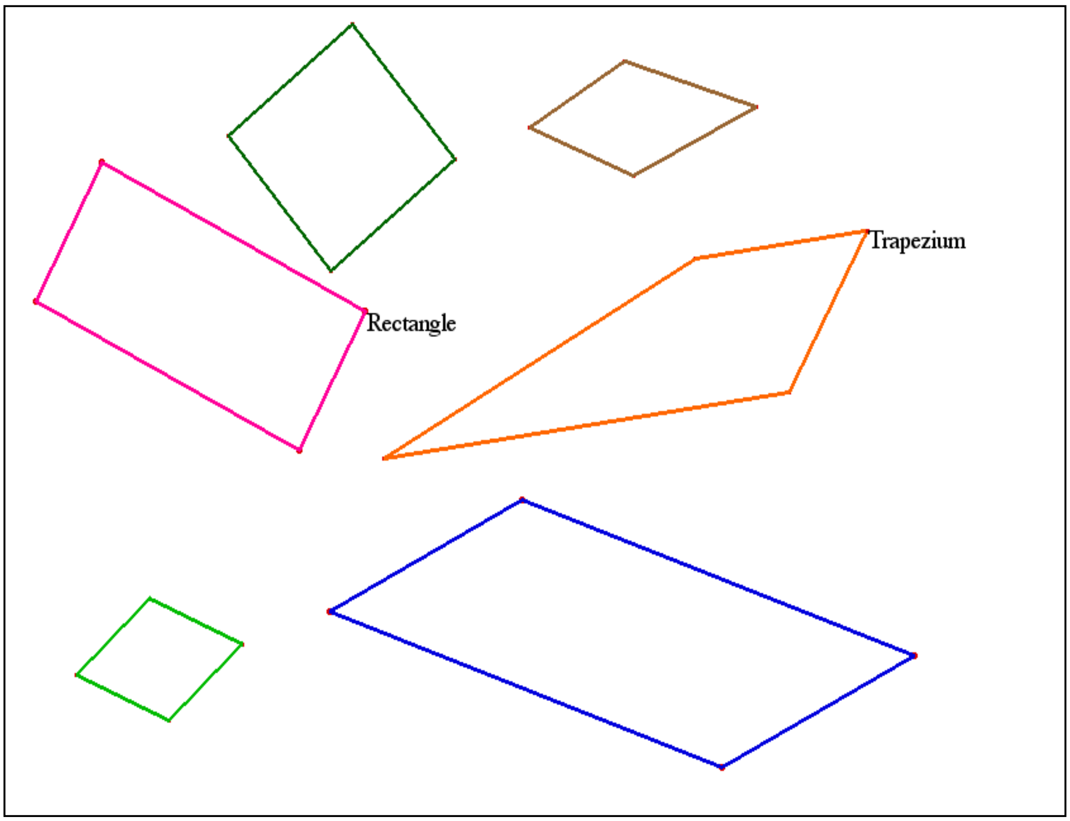

Fig. 1: Quadrilateral microworld developed for Session 1

PROCESSES OF TEACHING AND LEARNING

In this section of the paper we present a brief description of each lesson.

Session 1 — playing with the quadrilateral microworld, introducing the idea of properties

The first lesson of the design initiative started with the teacher (Pat) working in the computer suite, with all pupils sitting as a class in the middle of the room paying attention to the interactive whiteboard. On a flipchart at the front of the class Pat had written the words: Quadrilateral, Square, Rectangle, Parallelogram, Rhombus, Kite and Trapezium (these words were also on paper by each computer). Pat showed pupils (on the interactive whiteboard) how to move the quadrilaterals, how to colour in a quadrilateral and how to label it with its name, introducing this activity in the following way:

> and we've got here set up for you on the computers..special new software..it's brand new to our school..it's been brought in specially..it's been adapted and created for you..and here we've got a collection of quadrilaterals…4 sided shapes.. I'm going to show you how to use this… up here we've got a toolbar..most of that toolbar for the first task you don't need to know about……. right these shapes here you've got to know the names of them…does anyone know what that shape is called

As Pat pointed to each particular quadrilateral she asked the pupils to name it and there was always at least one pupil who was able to do this. Pat then manipulated each quadrilateral on the interactive whiteboard, emphasising that it always stays the same shape "it stays a kite all the time". She then showed the pupils how to name each quadrilateral on the screen and finally said:

> your task is to go to your computer and learn two things……learn….find out to move the shapes..how to move them around and to make them bigger..and the 2nd task is to write the name of each shape by the side of it…has anyone got any questions.

Analysis of the video recording shows that two of the pupils, Zacharias and Michael[1] , spent considerable time moving each individual quadrilateral and also filling them with colour. Although they did not talk extensively as they moved the quadrilaterals on the screen they used language such as:

> what was that…oh yes a square..it doesn't look like a square now (as they moved the square);
>
> how can we make it a square, that's a square…what was it..a rectangle..(as they moved the rectangle);
>
> how do you make it bigger again;
>
> you could just make it a line;
>
> that looks like a clock (rotating the sides of a trapezium);
>
> let's make a bumper;

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[1] Two pairs of pupils were video recorded during each lesson, one higher attaining pair and one lower attaining pair. These were obtained by asking the teacher to rank the pupils according to their attainment in class and we then divided this list into thirds. Chantale was chosen from the top third of ranked pupils and she worked with Candice throughout the 6 sessions. Zacharias was chosen from the bottom third and he worked with Michael throughout the six session. Chantale had been targeted, by the class teacher, to obtain Level 4 in the Stage 2 mathematics tests and Zacharias had been targeted to obtain Level 3c.

> this is a boomerang.

Their spoken language gives an indication of where they were focusing their attention and the actions they were carrying out. They were beginning to notice, for example, that a rectangle can be transformed into a square. Sometimes their language referred to the world outside mathematics and sometimes to the process of transforming a shape. Zacharias and Michael did not name any of the quadrilaterals until the teacher explicitly intervened and told them to do this. By the end of the lesson they had named three of the quadrilaterals. Their spoken language, although not extensive, was focused on the task.

Another pair, Chantale and Candice, approached the activity rather differently. They first filled in all the quadrilaterals with colour and then very systematically (and correctly) labelled each quadrilateral, paying considerable attention to spelling. It was only after this process they then started to manipulate each quadrilateral saying things like:

> that looks like a kite now (as they moved the general quadrilateral);
>
> that looks like a parallelogram (as they moved the rhombus).
>
> move the rectangle…..you click and pull,,,you click and pull
>
> it's too skinny (as rectangle width is decreased)
>
> that looks like a parallelogram (as move kite)
>
> do it tiny
>
> looks like a shoe (as trapezium is moved)
>
> that one's too skinny thanks to you
>
> I want it skinny
>
> that's not a square..it's a mouse head (as they make the square into a dot)
>
> that's too fat..it's like train tracks (as they move the parallelogram)

Their spoken language suggests that they were mostly focused on relating the mathematical shapes to objects in the world outside mathematics.

Approximately fifteen minutes before the end of this lesson the teacher called the class back together for a final plenary session, saying:

> I was so impressed and so excited ..you did it much quicker and everyone in the whole class managed..for what I could see nearly all of you were able to label them accurately..I can see that all of you could label the shapes..now can anyone tell me anything special..any special properties..any special things about a square…any special qualities it has..it always has..

This focus on the properties of quadrilaterals elicited the following types of responses:

(for a square) — all sides the same length, it is a quadrilateral;

(for a rectangle) — not all the sides are equal, a four sided shape, its longer than a square;

(for a parallelogram) — they are diagonal..the sides are diagonal, the sides are like train tracks, there's two parallel lines on there;

(for a rhombus) — like a square, if you squash a square you get a rhombus, they are parallelish;

(for a trapezium) — like a triangle with its top off.

This first session ended with pupils returning to the computers to make any picture they liked with the quadrilateral microworld, an activity which did not engage them as much as the previous one had done.

Session 2 — noticing and writing about properties of quadrilaterals

Pat started this lesson by working with the whole class:

> this is the 2nd week of 6 weeks and our objective by the end of the 6 weeks…you will be able to recognise shapes…you will be able to name them and you will be able to tell me and other people some of the properties of those shapes….how we can describe a shape..what makes it different from other shapes

Pat had written the names of the quadrilaterals on a flip chart and had made cardboard cut-outs of each quadrilateral. She picked one of these and asked a pupil to place this on the flip chart next to its name. As they did this she said:

T can we all say that word

All trapezium

This process was repeated for each quadrilateral. Pat then introduced them to what she called 'new mathematical vocabulary'. The following words had been written on the board (and were also on paper besides each computer): opposite, equal, parallel, adjacent, angle, sides, right, diagonal. Pat went through each word in turn explaining its meaning, sometimes bringing pupils up to the front of the class (for example to demonstrate opposite, and adjacent) and sometimes holding up pieces of card (for example to demonstrate parallel lines). However, as we shall see later, some of this activity seems to have caused pupils to become confused about the mathematical meaning of opposite and adjacent.

After this flip chart activity Pat moved to the interactive whiteboard saying:

> look at the screen…we've got a rectangle and a parallelogram (Pat moves the shapes and emphasises that all the time they stay the same shape)…..now can somebody describe to me..lets look at the rectangle first…who can tell me anything about a rectangle which is special to it…Zacharias

opposite sides are the same size (response from Zacharias)

Pat then showed the pupils how they could write the properties of a quadrilateral on the screen. They then worked in pairs at the computer continuing with this activity (see for example, Figure 2). Most pupils did write something on the screen (Table 1). Some of this was mathematically correct and some incorrect. Some phrases are difficult to make sense of, for example 'all sides are equal' for a parallelogram and a rectangle, but we think the pupils meant by this 'each side has a side which is equal to it'. Some phrases are non-mathematical (at least in this context), for example ' it's the colour turquoise' and some relate to transformations, for example 'it can be a diamond'.

Pupil pair, categorised by the SATs level predicted by the class teacher	**Pupils' writing (on screen) for 1st quadrilateral**	**Pupils' writing (on screen) for 2nd quadrilateral**
Level 4	(parallelogram) 2 parallel sides, opposite sides, 4 vertices, 4 right angles	(trapezium) 4 vertices, 3 right angles, no equal sides
Level 4	(square) all sides are equal, 4 right angles, 4 vertices, opposite	(kite) it has 4 vertices, has opposite sides, each side is equal
Level 4	(square) all four sides are equal, has four right angles, all sides are opposite	(kite) two of the long sides are equal
Level 4	(parallelogram) it has	(rhombus)
Level 4	(parallelogram) it has 4 sides, they are like train tracks, they are parallel, all sides are equal, it doesn't have any right angles, it's the colour turquoise, it can be a diamond	(rectangle) 4 sides, all sides equal, 2 sides are long and 2 sides are short, it's the colour red, it is not parallel, there are 4 right angles, it is not diagonal, it has opposite sides, it can be a 3D shape as well
Level 3a	(square) it's got 2 diagonal lines, it has 2 eqeul (sic)lines	(kite) a kite has 2 lines the same length, it's got 2 diagonal lines
Level 3a	(parallelogram) it has four sides and vertices, the sides are parallel	(rectangle) all sides are adjacent, opposite sides are equal, opposite sides are parallel, 4 vertices
Level 3b,3c	(parallelogram) the parallel opposite and it can be a square and a kite and four	(rectangle) the rectangle is opposite and got four sides and it can be…
Level 3b, 3c	(square)	(rectangle)
Level 3b, 3c	(square)	(kite)
Level 3b, 3c	(parallelogram) opposites, parellels (sic), diagonal, two pares (sic) of equal sides	(rectangle)

Table 1: Pupils' writing on the screen in response to the questions 'What is the same and what is different?'

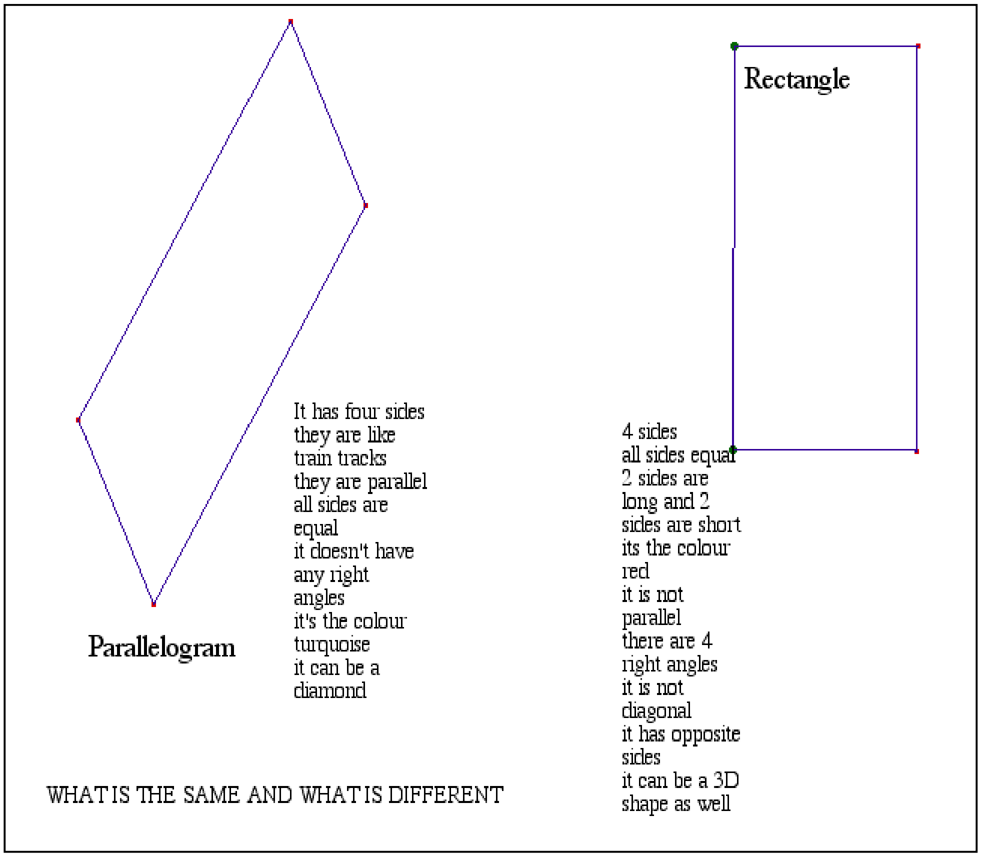
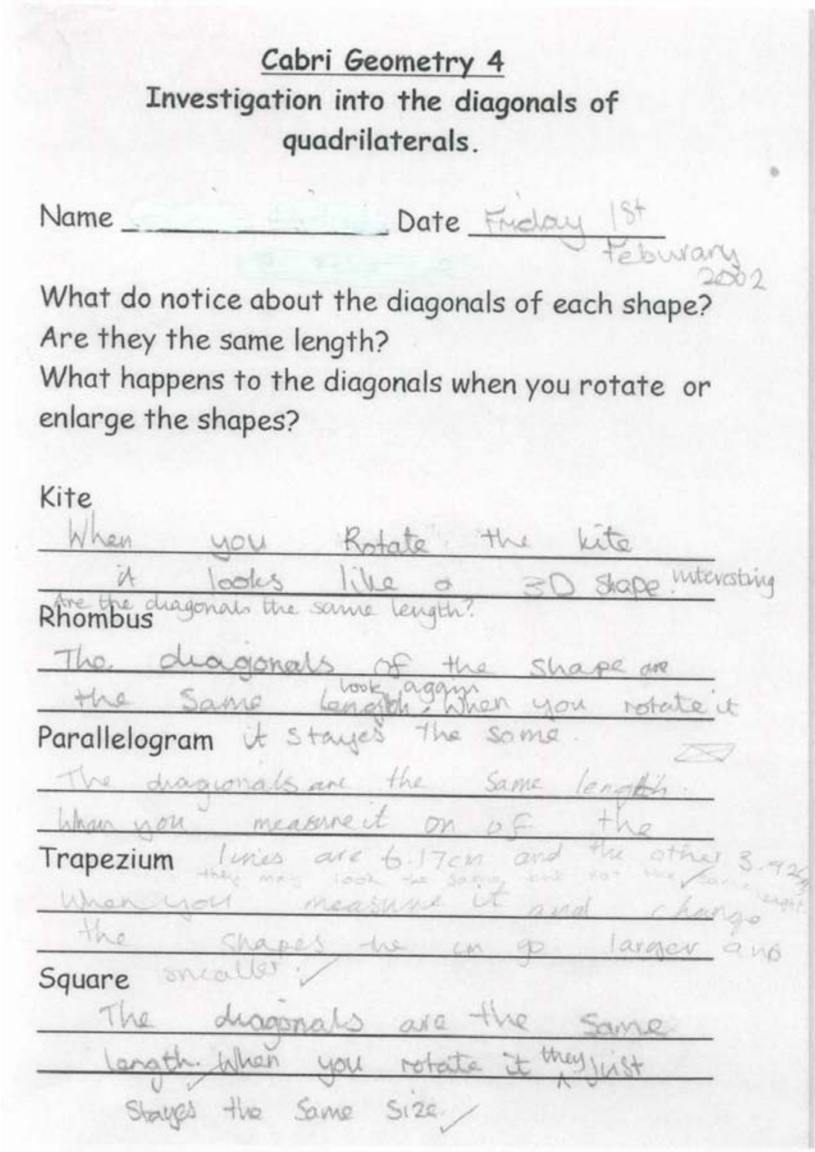


Fig. 2: Similarities and differences between a parallelogram and a rectangle: work produced by one pair of pupils in Session 2

Session 3 — investigate diagonals of quadrilaterals

In this session the focus of attention shifted from the properties of the sides of quadrilaterals to the properties of diagonals. Pupils were shown how to measure the length of a line segment, with the intention that they would use these measurement to notice the invariant properties. However these measurements often cluttered up the screen and were not always easy to make sense of. The pupils were asked (individually) to write their findings on paper during this session (See Fig. 3a) and an overview of what was written down is presented in Table 2. It would appear that the lower attaining pupils wrote less on paper than they had written on the computer screen in the previous lesson (Table 1). The vast majority of pupils did not pay attention to any angle properties (for example opposite angles of a rhombus are identical) apart from right angle properties. The teacher had explicitly told the pupils to check for right angles by holding up the corner of a book to the screen.



Cabri Geometry 4

Investigation into the diagonals of quadrilaterals.

Name [illegible] Date Friday 1st Feburary 2002

What do notice about the diagonals of each shape?
Are they the same length?
What happens to the diagonals when you rotate or enlarge the shapes?

Kite

When you Rotate the kite it looks like a 3D Shape interesting
Are the diagonals the same length?

Rhombus

The diagonals of the Shape are the Same length look again When you rotate it it stayes the same

Parallelogram

The diagonals are the Same length When you measure it on of the lines are 6.17cm and the other 3.92

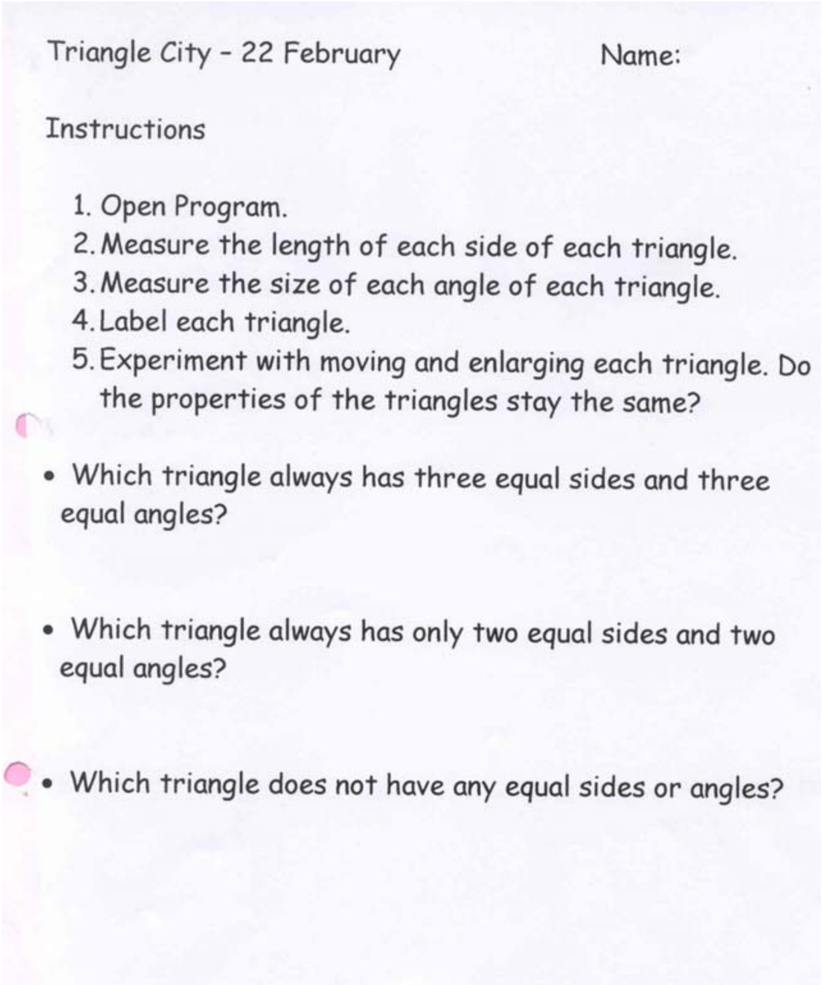
Trapezium

they may look the same but not the same length
When you measure it and change the Shapes the can go larger and smaller ✓

Square

The diagonals are the Same length. When you rotate it they just Stayes the Same Size ✓

Fig. 3a



Triangle City - 22 February Name:

Instructions

1. Open Program.
2. Measure the length of each side of each triangle.
3. Measure the size of each angle of each triangle.
4. Label each triangle.
5. Experiment with moving and enlarging each triangle. Do the properties of the triangles stay the same?

- Which triangle always has three equal sides and three equal angles?
- Which triangle always has only two equal sides and two equal angles?
- Which triangle does not have any equal sides or angles?

Fig 3b

Pupil	Square	Rectangle	Parallelogram	Rhombus	Kite	Trapezium
Level 4	they are the same length				they are not the same length. They cross at right angles	
Level 4					the diagonal lines are not the same length as the other	
Level 4	the lines are always the same length		one diagonal line is longer than the other		one line is always longer than the other	
Level 4	the diagonals are the same length. When you rotate it just stays the same			the diagonals of the shape are the same length. When you rotate it stays the same	when you rotate the kite it looks like a 3D shape	they may look the same but not the same length. when you measure it and change the shapes it go larger and smaller
Level 4			they don't have to be the same size as each other	one diagonal is bigger than the other	one diagonal is bigger than the other. they cross in right angles	
Level 4					they are not the same length they cross at right angles	
Level 3a	the diagonal lines of a square are always the same length and is a rectangle		the diagonals of a parallelogram are not parallel		ther not the same length they cross at length	the lines of a trapezium are not the same length
Level 3a					when you rotate the kite it looks like a 3-D shape	
Level 3a	the diagonals are the same length				they are not the same length, they cross at right angles	
Level 3b,3c				the length is different	the length is different	

Table 2: Pupils' writing on paper in response to questions related to the properties of diagonals of quadrilaterals.

Session 4[2] — investigate parallel and perpendicular lines

This session focused on parallel and perpendicular lines. The pupils were shown how to construct parallel and perpendicular line in Cabri and the session was very much an exploratory one for the pupils in which they experimented with these constructions,

Session 5 — play with triangle microworld and investigate properties of triangles

In this lesson the normal teacher (Pat) was not present and the lesson was taught by the

ICT support teacher, provided by the LEA Education Action Zone. This teacher was working in the school each Friday throughout this academic year, in order to support all teachers to incorporate ICT into their practice. The aim of the session was for pupils to investigate the properties of particular triangles (equilateral, right angled, isosceles, scalene). In contrast to what was normally prepared for this lesson a very structured worksheet was presented to the pupils (See Fig 3b). Pupils were also introduced to the idea of measuring angles at the beginning of this session. From the beginning the majority of pupils did not engage with the presented activity. They quickly rushed through the questions, focusing on measuring angles, which seemed to detract from a focus on the properties of triangles. After a short period of time the pupils started to put their hands up saying things like "miss I've finished". This lesson contrasts with the other lessons, and shows that there is nothing inherent in a particular software (the researchers had prepared a triangle microworld for this session which was similar to the quadrilateral microworld used in other sessions).

Session 6 — black box game, three questions to guess which quadrilateral/triangle is in my box; construction of rectangle or parallelogram in Cabri

At the beginning of this, the final session, Pat introduced the class to a black box activity:

T I'm going to start off today with a game in my box here I have some shapes and you are going to guess what the shape is…if you guess within 5 guesses a point you..if you don't guess within 5 guesses..a point to me..now you will ask questions to which I can only answer yes or no…the first three questions you can't say is it a square..is it a triangle…is it a rhombus..is it an iscosceloes triangle…you've got to ask questions thinking about the properties of the shape..right…for example..lets begin..I've got a shape here..ask me questions about my shape…you're allowed 5 questions and you keep a tally mark of the no of questions we've had..a trial run …next question..Zacharias

A is it a quadrilateral

T yes ..it is a quadrilateral

[2] Only a brief summary of this session is presented here as the video recording has not yet been analysed in detail.

T who'd like to ask the next question…Emma

E has it got parallel sides

T has it got parallel sides..yes it has.. so you need to be thinking a shape that's got parallel sides..a shape that's a quadrilateral and a shape that has not got 3 sides…can you think of a question to do with the angles…would you like to guess a shape now..rasheen

P a square

T it is a square..the opposite sides are parallel..it has got 4 sides…they are all the same length and each angle is 90 degrees (T hold up square shape from box)

This activity which was carried out without computers showed that the pupils were developed an awareness of properties of quadrilaterals as illustrated by the following questions from the pupils:

has it got an acute angle?

is it an isosceles triangle?

has it got any right angles?

has it got equal sides?

Confusion related to the meaning of adjacent sides, which the analysis of the data shows was emerging in the second session, was evident in this final session, as the following interchange shows:

Emma has it got any adjacent sides?

T has it got adjacent sides…they've all got adjacent sides..all got sides next to each other

and then later in the same session

Pupil are the sides adjacent?

T all the sides are adjacent ..every shape we've got whether it is a triangle or a quadrilateral..that side is adjacent to that one

It seems that whatever meanings the pupils originally constructed for the word adjacent in Session 2, no amount of 'telling' by the teacher was impacting on these meanings. (The question in the post initiative diagnostic assessment which asked 'Are adjacent sides equal' for a range of shapes was answered incorrectly by the majority of pupils).

After this activity Pat worked at the interactive whiteboard and showed pupils how to construct a rectangle for themselves. She then asked pupils to work in pairs and construct either a rectangle or a parallelogram. The pupils found this relatively difficult, but some of them, with support, were able to successfully construct quadrilaterals for themselves.

A DISCUSSION OF LEARNING OUTCOMES

Analysing the processes of teaching and learning is still ongoing. This involves analysing the video data, pupils' work on the computer and on paper, the diagnostic assessment and the pupil and teacher interviews. We are confident that the dynamic geometry environment has enabled pupils to enter the mathematical world of properties of polygons and that manipulating geometrical shapes at the screen has been key in this respect. This manipulation helps them pay attention to similarities and differences between properties of different shapes. However the data also shows that the way in which the teacher orchestrates the lesson through talk and interaction with pupils is also key. At the beginning and end of each lesson Pat directed the pupils to pay attention to the naming of quadrilaterals and the properties of quadrilaterals and she did this through her own use of language and through eliciting pupils' ideas. When pupils worked in pairs at the computer what they did was a creative transformation of what they have observed Pat doing and what Pat has told them to do. However because each pupil brings their own history of learning to any new learning situation within any particular classroom each pupil works differently when they work individually.

Analysis of the data suggests that the seeds of what pupils say themselves can be found within Pat's language and gesture and what was written for the pupils on paper. The antecedent's of pupils' ongoing confusion related to the mathematical meaning of adjacent can be found in the way this idea was introduced to pupils in Session 2. On one of the worksheets presented to pupils were the following questions "What have you found out about the sides of each shape? Are any sides the same length? Are they opposite sides? Are they adjacent sides? Are any sides parallel?" The words 'parallel' and 'equal length' relate to properties, but the words adjacent and opposite describe mathematical states. They are qualitatively different types of words and using all phrases together seems to have confused the pupils with respect to the meaning of adjacent and opposite.

How the teacher supports pupils to engage in the mathematical problem being solved is key to the whole process of learning mathematics. Within this work pupils did engage throughout most of the lessons, apart from lesson 5 and here our analysis suggests that pupils did not engage in this activity because it was too closed and directive. The numbering of the activities also seemed to push them to rush to finish as opposed to engaging with the problem. In addition the worksheet placed too much emphasis on the technicalities of measuring angles and not enough emphasis on the mathematical properties of triangles.

Careful design of both the starting point activity presented by the teacher and also the mathematical microworld developed in Cabri also seems to be important in focusing pupils' attention on mathematical activity. Once they start to see the quadrilaterals on the screen, through the mathematical framework of properties of quadrilaterals, they are then able to say and write what they see. This 'saying what they see' and 'writing what they say' also appears to be important in terms of their mathematical learning. Writing on the screen appears to be easier than writing on paper for the lower attaining pupils. The pupils' writing also gives the teacher feedback on their developing conceptions, as does their discussions at the beginning and end of each lesson. The work with the interactive whiteboard allows the whole class to attend to mathematical properties and helps to create a community of inquirers. In this sense the work with the interactive whiteboard allows pupils to share and communicate their ideas and collectively focus their attention on the same dynamic environment which they are also able to use for themselves.

Analysis of the data suggests that what pupils do themselves when they work at the computer is a transformation of what the teacher has demonstrated. Pat's lack of experience with Cabri meant that sometimes the way she demonstrated how to do something with the software, was not the best way from the point of view of the transparency of the mathematics. For example in the last lesson she used lines to construct a rectangle, as opposed to line segments and this meant that the constructed rectangle could not be manipulated on the screen. All the pupils followed this way of constructing when they worked on their own, as they were modelling what the teacher had presented.

Pupils spontaneously began to pay attention to the hierarchy of polygons, through observation of the way in which one triangle can be transformed into another triangle, for example, a rectangle into a square. The teacher did not explicitly draw on these ideas in the plenary sessions, although this will become more of a focus within the revised design initiative.

Several pupils in the class did not learn as much as the others, and these were mainly pupils who were being withdrawn in order to prepare for an examination for a local independent school which offers scholarships to pupils, and pupils who had particular behavioural difficulties such as attention deficit disorder.

In summary, analysis of the data so far suggests that pupils were learning:

- to use dynamic geometry as a tool for doing mathematics
- to become aware of mathematical properties of quadrilaterals and triangles and to talk about these properties
- to shift attention from extra-mathematical to intra-mathematical meaning

- to identify and name particular quadrilaterals
- the mathematical meaning of diagonal
- the idea of a polygon

Is it as simple as pupils learn from what they pay attention to? Certainly manipulating geometrical shapes in a dynamic environment supported pupils to pay attention to what changes and what stays the same when you manipulate a quadrilateral. This is a non trivial idea and relates to the invariant properties of a quadrilateral (for example a rectangle always has: opposite sides equal; opposite sides parallel: 4 right angles; equal diagonals). An alternative approach would be to expect pupils to memorise these properties, but memorising lists of properties does not relate to an understanding of what these properties mean. A square is a square because of its invariant properties. A parallelogram is a parallelogram because of its invariant properties. This **is** the world of mathematics and these Year 6 pupils were clearly interested in learning about these ideas, ideas which are part of the National Numeracy Strategy.

REWORKING THE DESIGN

A key part of the InterActive Education project is that each teacher is expected to rework their design initiative in order to teach a new group of pupils in the subsequent year of the project. Teachers and researchers are learning from viewing the video recordings and from the analysis of the data. The design initiative reported in this paper will be reworked and taught again to a new group of Year 6 pupils in November 2002. From watching the video recordings Pat has become very aware of the subtle ways in which pupils transform language and activity for themselves (for example 'like train tracks' for parallel lines becomes 'short sides of a rectangle cannot be parallel because they are too far apart'). This is evidence of pupils as active constructors of knowledge. In the revised design initiative, to be taught in November 2002, Pat plans to make more use of writing on the screen (for example Fig. 2) by printing out this work for pupils to annotate and comment on by hand within lessons in the normal classroom. In other words the computer printouts will be resources for them to use for their ongoing learning and reflection[3].

Pat had never used the computer with pupils before the work reported in this paper, although she is a very experienced primary school teacher and the mathematics coordinator in her school. This paper is part of the reflection and communication process. We do not expect other teachers to copy the idea intact, but we do think that they may be able to creatively transform it for themselves. A similar support mechanism for this type of professional development could be created by teachers

[3] This will take an enormous amount of organisation as printing pupils' work is non trivial in the particular network set up within this school.

working collaboratively within their own school or their Local Education Authority. However this would involve recognising the nature of the support, which involved dialogue, feedback and iteration, starting with a teaching focus which related to the teacher's own concerns. This approach requires people time and could not be devolved to a computer-based system. It also involves recognising how much is still to be learned in the area of teaching and learning mathematics.

REFERENCES

Brown, L & Coles, A. (2001) Natural Algebraic Activity in Proceedings of the 12th ICMI Study Conference *The Future of the Teaching and Learning of Algebra*, vol 1, 120-127pp, H. Chick, K. Stacey, Jill & John Vincent (eds).

Clements D & Battista M (1992), Geometry and Spacial Reasoning, in D. A. Grouws (ed), Handbook of Research on Mathematics Teaching and Learning, NCTM, Macmillan Pub. Company.

Cole, M & Engestrom, Y. (1993) A Cultural-historical approach to distributed cognition, Salomon, G. (ed) *Distributed Cognition*, CUP, Cambridge.

Crook, C. (1994) *Computers and the Collaborative Experience of Learning*, Routledge, London.

Davis, B., Dennis, S. & Luce-Kapler, R. (2000) *Engaging Minds: Learning and Teaching in a Complex World*. Lawrence Erlbaum Publishers, London.

Salomon, G. (1993) *Distributed Cognitions*, *Psychological and Educational Considerations,* 88-110. Cambridge: Cambridge University Press.

Sutherland, R. & Balacheff, N. (1999) Didactical Complexity of Computational Environments for the Learning of Mathematics, the *International Journal of Computers for Mathematical Learning* Vol. 4, pp 1-26

van Hiele, P.M. (1959/1985) The child's thought and geometry. in D. Fuys, D. Geddes, & R. Tischler (eds). *English translation of selected writings of Dina van Hiele-geldorf and Pierre van Hiele* (pp 243-252), Brooklyn, NY, Brooklyn College School of Education

Wertsch, J. (1991) *Voices of the Mind: A Sociocultural Approach to Mediated Action*, United States of America: Harvester Wheatsheaf

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